

Calculation and optimization of laser acceleration in vacuum

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Extraordinarily high fields generated by focused lasers are envisioned to accelerate particles to high energies. In this paper, we develop a new method to calculate laser acceleration in vacuum based on the energy exchange arising from the interference of the laser field with the radiation field of the particle. We apply this method to a simple accelerating structure, a perfectly conducting screen with a round hole, and show how to optimize the energy gain with respect to the hole radius, laser angle, and spot size, as well as the transverse profile of the laser. Limitations and energy scaling of this acceleration method are also discussed.

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I. INTRODUCTION

Acceleration of charged particles by laser fields in vacuum can be calculated as

$$\Delta U_{\text{acc}} = e \int \mathbf{E} \cdot \mathbf{v} dt, \quad (1)$$

where ΔU_{acc} is the energy gain, e is the charge, \mathbf{E} is the electric field, \mathbf{v} is the particle's velocity, and the time integral is taken along the particle's path. In a straight trajectory approximation, when \mathbf{v} in Eq. (1) is considered as a constant unperturbed velocity, according to the Lawson-Woodward theorem [1], laser acceleration in vacuum is possible only in close proximity to material boundaries. The acceleration occurs because currents and charges induced by the laser field in the material distort the incident electromagnetic field in a way which gives a nonzero value for the integral.

A direct calculation of the integral in Eq. (1) requires solving Maxwell's equations in the vicinity of the material boundaries. In most cases, this leads to a formidable electromagnetic problem and requires extensive numerical computations. Only very simple geometries allow an analytical calculation of the energy gain directly from Eq. (1) (see, e.g., [2,3]).

In this paper we develop a new method to calculate the energy gain ΔU_{acc} . It is based on the energy balance equation for the electromagnetic field energy and the particle's energy, and only requires knowledge of the radiation field in the far zone. In its most general formulation, it is not limited to vacuum and straight trajectories. It can also be used for acceleration in a medium (e.g., inverse Cherenkov acceleration), and curvilinear trajectories (such as in inverse free-electron laser (FEL) acceleration).

To demonstrate advantages of the new method, we apply it to a relatively simple problem: laser acceleration

of a particle passing through a round hole in a perfectly conducting metal screen. The assumption of perfect conductivity of the metal is valid if the laser frequency is smaller than the plasma frequency for the metal. Two different laser illuminations are considered: first with a higher-order laser mode, and second with two crossed Gaussian laser beams. Note that crossed Gaussian lasers are used in the LEAP experiment at Stanford University [4] and in the proposed E-163 experiment at SLAC [5]. In the limit when the hole radius a tends to zero, we show that our result agrees with direct calculation of the integral (1). Taking into account the effect of a damage threshold for materials, we show how our calculations also allow optimization of the energy gain for given laser parameters and find the limits of this acceleration method.

II. RELATION BETWEEN RADIATION FIELDS AND ENERGY GAIN

Consider a bunch passing through a hole in a perfectly conducting metal screen, as shown in Fig. 1. The hole may have an arbitrary shape, although in subsequent sections we assume that it is round, with a radius a . At the time of passage, the bunch is irradiated by a laser pulse, and due to the interaction with the laser light, particles in the bunch are accelerated or decelerated depending on the phase of the laser wave.

We introduce a surface of large radius R enclosing a volume V which includes the acceleration area. Eventually, we take the limit $R \rightarrow \infty$. Initially, at $t \rightarrow -\infty$, a particle in the bunch and the laser pulse are located outside of the surface S . After the interaction, when $t \rightarrow \infty$, they leave the volume V .

We use the energy balance equation for the electromagnetic field (see, e.g., [6,7]):

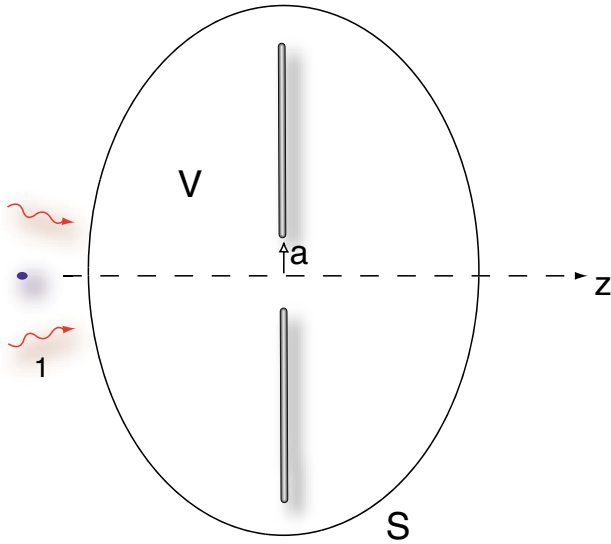


FIG. 1. (Color) Layout of a vacuum laser acceleration experiment. A perfectly conducting screen with a round hole of radius a is located at $z = 0$. Initially, a particle and a laser pulse are at position 1 outside of the volume V .

$$\frac{\partial}{\partial t} \int_V dV \frac{E^2 + H^2}{8\pi} + \int_V dV \mathbf{j} \cdot \mathbf{E} = - \int_S \mathbf{S} \cdot \mathbf{n} dS,$$

where the integration goes over the volume V with the surface boundary S , \mathbf{n} is a unit vector in the outward direction normal to the surface, \mathbf{j} is the current density, \mathbf{S} is the Poynting vector, $\mathbf{S} = (c/4\pi)\mathbf{E} \times \mathbf{H}$, and \mathbf{H} is the magnetic field. Integrating this equation over time, from $t = -\infty$ to $t = \infty$, and taking into account that at $t = \pm\infty$ there is no electromagnetic field inside the volume V , we find

$$\int_{-\infty}^{\infty} dt \int_V dV \mathbf{j} \cdot \mathbf{E} = - \int_{-\infty}^{\infty} dt \int_S \mathbf{S} \cdot \mathbf{n} dS.$$

The current density \mathbf{j} includes the current in the metal and the current associated with the moving point charge, $\mathbf{j} = ev\delta(\mathbf{X}(t))$, where $\mathbf{X}(t)$ is the particle's trajectory. The former, however, does not contribute to the integral $\int dV \mathbf{j} \cdot \mathbf{E}$, because this integral is equal to the energy deposited inside the metal due to the Ohmic heating, which is neglected in the limit of perfect conductivity. Hence the integral reduces to $e \int_{-\infty}^{\infty} \mathbf{v} \cdot \mathbf{E} dt$ taken along the trajectory. It is equal to the energy gain (or loss, if negative) ΔU of the particle due to the interaction with the field. Hence

$$\Delta U = - \int_{-\infty}^{\infty} dt \int_S \mathbf{S} \cdot \mathbf{n} dS. \quad (2)$$

Note that this formula is exact and is valid for arbitrary curvilinear motion of the particle under the influence of an external field.

In the calculation of fields, it is convenient to use the Fourier transform, which we define as

$$\left\{ \begin{array}{l} \mathcal{E}(\omega) \\ \mathcal{H}(\omega) \end{array} \right\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{i\omega t} \left\{ \begin{array}{l} \mathbf{E}(t) \\ \mathbf{H}(t) \end{array} \right\}. \quad (3)$$

Using Parseval's theorem we find

$$\begin{aligned} \Delta U &= -\frac{c}{2} \int_{-\infty}^{\infty} d\omega \int_S \text{Re}[\mathcal{E}(\omega) \times \mathcal{H}^*(\omega)] \cdot \mathbf{n} dS \\ &= -\frac{c}{2} \int_{-\infty}^{\infty} d\omega \int_S \text{Re}(\mathcal{E}(\omega) \cdot \mathcal{E}^*(\omega)) dS, \end{aligned} \quad (4)$$

where the asterisk denotes complex conjugate, and we use the relations $\mathcal{H} = \mathbf{n} \times \mathcal{E}$ and $\mathbf{n} \cdot \mathcal{E} = 0$, valid in the far zone.

The field entering Eq. (4) is a superposition of the laser field, \mathcal{E}^{LS} , and the particle's field, \mathcal{E}^{PS} :

$$\mathcal{E} = \mathcal{E}^{\text{LS}} + \mathcal{E}^{\text{PS}}, \quad (5)$$

where the letter “S” in the superscript indicates that these are the fields in the presence of the screen. Substituting Eq. (5) into Eq. (4) and employing the notation $\Delta U(\mathcal{E}_1, \mathcal{E}_2) \equiv -\frac{c}{2} \int_{-\infty}^{\infty} d\omega \int_S \text{Re}(\mathcal{E}_1(\omega) \cdot \mathcal{E}_2^*(\omega)) dS$ we obtain several terms. The term $\Delta U(\mathcal{E}^{\text{LS}}, \mathcal{E}^{\text{LS}})$ corresponds to the integrated energy flow of the laser light through the surface S without the beam. This term vanishes because we assume that there are no losses in the screen and hence the incoming laser energy is equal to the outgoing one. The term $\Delta U(\mathcal{E}^{\text{PS}}, \mathcal{E}^{\text{PS}})$ describes the energy radiated by the particle passing through the hole in the screen when there is no laser field. This term scales as a square of the particle's charge and is not relevant to the acceleration of a single particle considered in this paper. Only the cross term,

$$-c \int_{-\infty}^{\infty} d\omega \int_S \text{Re}(\mathcal{E}^{\text{LS}} \cdot \mathcal{E}^{\text{PS}*}) dS, \quad (6)$$

is responsible for acceleration of the particle.

In what follows, we need a notation for the laser field without the screen, \mathcal{E}^{L} , and the beam field without the screen, \mathcal{E}^{P} . We define the radiation fields \mathcal{E}^{LR} and \mathcal{E}^{PR} as a difference between the field with the screen and the field in free space: $\mathcal{E}^{\text{LR}} = \mathcal{E}^{\text{LS}} - \mathcal{E}^{\text{L}}$, $\mathcal{E}^{\text{PR}} = \mathcal{E}^{\text{PS}} - \mathcal{E}^{\text{P}}$. The radiation fields are generated by currents flowing in the screen. The fields \mathcal{E}^{LS} and \mathcal{E}^{PS} can be considered as a superposition of the radiation fields and the fields without the screen, i.e.,

$$\mathcal{E}^{\text{LS}} = \mathcal{E}^{\text{L}} + \mathcal{E}^{\text{LR}}, \quad \mathcal{E}^{\text{PS}} = \mathcal{E}^{\text{P}} + \mathcal{E}^{\text{PR}}. \quad (7)$$

For calculation, it is convenient to cast Eq. (6) into a different form. Using the second expression in Eq. (7) we represent Eq. (6) as a sum of two terms. The first one involves the particle's field without the screen:

$$-c \int_{-\infty}^{\infty} d\omega \int_S \text{Re}(\mathcal{E}^{\text{LS}} \cdot \mathcal{E}^{\text{P}*}) dS.$$

This term describes interference of the charge's Coulomb field in vacuum with the laser field. In the limit $R \rightarrow \infty$, this term vanishes because the Coulomb field moves with the charge with velocity $v < c$, and the laser light propagates with the speed of light c . Since we assume that the laser pulse overlaps with the particle in the vicinity of the hole, at a large distance from the hole these two fields are separated in space. Hence, the particle's acceleration is given by the second term:

$$\Delta U_{\text{acc}} = -c \int_{-\infty}^{\infty} d\omega \int_S \text{Re}(\mathcal{E}^{\text{LS}} \cdot \mathcal{E}^{\text{PR}*}) dS, \quad (8)$$

for which we use the notation ΔU_{acc} . Notice that the presence of the field \mathcal{E}^{PR} in this equation indicates that a particle can be accelerated only if it radiates.

Although in the above derivation we refer to the layout of the acceleration experiment outlined in Fig. 1, our result is not limited by this specific arrangement. With a slight modification, it can also be used for calculation of the energy gain for other acceleration schemes, such as, e.g., inverse FEL or inverse Cerenkov acceleration. The close relationship between acceleration and radiation has been explored in Refs. [1,8]. Recently, Eq. (8) is also derived by Xie [9].

In our calculations of the radiation field below we assume that the particle moves with a constant velocity. Hence, we neglect the effect of the laser field on the particle's trajectory, as well as the effect of radiation reaction. Such an approximation describes linear acceleration proportional to the laser electric field.

III. DIFFRACTION RADIATION ON A ROUND HOLE

Following the approach developed in the previous section, we first calculate the radiation field \mathcal{E}^{PR} of the particle. We now assume that the hole in the screen is round, with radius a , and consider a relativistic particle moving along the axis of the screen with a constant velocity v close to the speed of light. In the limit of the large Lorentz factor, $\gamma \gg 1$, the radial electric and azimuthal magnetic fields of the particle are

$$E_r^{\text{P}}(r, z, t) = H_{\theta}^{\text{P}}(r, z, t) = \frac{e\gamma r}{[r^2 + \gamma^2(z - vt)^2]^{3/2}}, \quad (9)$$

where $r = \sqrt{x^2 + y^2}$ is the radial distance.

To calculate the radiation field in the far zone we use diffraction formulas [7,10,11]. This approach is valid if the reduced wavelength of the radiation, $\lambda = \lambda/2\pi$, is much smaller than the radius of the hole a , and the diffraction angle is small. According to the diffraction theory [7], the field behind the screen, \mathcal{E}^{PS} , at large distance $R \rightarrow \infty$ and in the region $z > 0$, can be calculated by integration of the incident field \mathcal{E}^{P} on the screen at $z = 0$:

$$\mathcal{E}^{\text{PS}} = \frac{e^{ikR}}{R} \frac{i}{2\pi} \mathbf{k} \times \int_{\text{hole}} e^{-i\mathbf{k}\mathbf{r}} \mathbf{n} \times \mathcal{E}^{\text{P}} dS, \quad (10)$$

where $\mathbf{r} = (x, y)$ is the two-dimensional vector in the plane of the hole, \mathbf{k} is the wave number vector in the direction of the radiation, $k = |\mathbf{k}| = \omega/c$, and \mathbf{n} is the unit vector perpendicular to the surface of the hole. The integration in Eq. (10) goes over the cross section of the hole.

Equation (10) is derived in [7] for the case when the incident wave propagates in free space. In our problem the incident field is the Coulomb field carried by the particle. In this case, Eq. (10) gives the total field behind the screen including the field of the particle, and to find the radiation field, we need to subtract the Coulomb field of the electron. The latter can be calculated as the same integral in Eq. (10) in the limit $a \rightarrow \infty$, that is when the screen is removed. The result of such a subtraction is an integral, with the sign opposite to that in Eq. (10), in which the integration goes over the screen surface, rather than the hole [10]:

$$\begin{aligned} \mathcal{E}^{\text{PR}} &= \mathcal{E}^{\text{PS}} - \mathcal{E}^{\text{P}} \\ &= -\frac{e^{ikR}}{R} \frac{i}{2\pi} \mathbf{k} \times \int_{\text{screen}} e^{-i\mathbf{k}\mathbf{r}} \mathbf{n} \times \mathcal{E}^{\text{P}} dS. \end{aligned} \quad (11)$$

A more rigorous proof of this equation can be found in Ref. [11].

The particle's field on the screen is given by $E_r(r, 0, t)$ and $H_{\theta}(r, 0, t)$ in Eq. (9). Fourier transformation of these fields defined by Eq. (3) gives

$$\mathcal{E}_r^{\text{P}}(r, \omega) = \mathcal{H}_{\theta}^{\text{P}}(r, \omega) \approx \frac{ke}{\pi c \gamma} K_1\left(\frac{kr}{\gamma}\right), \quad (12)$$

where K_n ($n = 0, 1, 2, \dots$) is the modified Bessel function, and we have used $v \approx c$ in the above expression.

In the limit of large γ , the angle of the radiation relative to the z axis, θ , is small, $\theta \ll 1$. Substituting Eq. (12) into Eq. (11) and neglecting higher-order terms in θ , we find that \mathcal{E}^{PR} has the radial component only,

$$\begin{aligned} \mathcal{E}_r^{\text{PR}} &= -k \frac{e^{ikR}}{R} \int_a^{\infty} r dr \mathcal{E}_r(r, \omega) J_1(kr\theta) \\ &= -\frac{ek^2}{\pi \gamma c} \frac{e^{ikR}}{R} \int_a^{\infty} r dr K_1\left(\frac{kr}{\gamma}\right) J_1(kr\theta), \end{aligned} \quad (13)$$

where J_n ($n = 0, 1, 2, \dots$) is the Bessel function. The integration in the last formula can be carried out analytically [12],

$$\mathcal{E}_r^{\text{PR}} = A(\omega, \theta) \frac{e^{ikR}}{R}, \quad (14)$$

with

$$A(\omega, \theta) = \frac{e}{\pi \gamma c} \frac{ka}{\theta^2 + \gamma^{-2}} \left[\theta J_2(ka\theta) K_1\left(\frac{ka}{\gamma}\right) - \frac{1}{\gamma} J_1(ka\theta) K_2\left(\frac{ka}{\gamma}\right) \right].$$

This formula agrees with the rigorous solution of the diffraction radiation problem obtained in Ref. [13], if one takes the limit $\gamma \gg 1$, $ka \gg 1$ of their result. In the limit $\lambda \gg a\gamma^{-1}$ (but λ is still much less than a) we have [10]

$$A(\omega, \theta) = -\frac{e}{\pi c} \frac{\theta}{\theta^2 + \gamma^{-2}} J_0(ka\theta), \quad (15)$$

which in a small-angle approximation yields

$$A(\omega, \theta) = -\frac{e}{\pi c} \frac{\theta}{\theta^2 + \gamma^{-2}}. \quad (16)$$

Since the hole radius a drops out from the last equation, it is also valid in the limit $a \rightarrow 0$, when there is no hole in the screen. In this limit, it is usually called the transition radiation.

IV. ACCELERATION BY A HIGHER-ORDER LASER MODE

For the laser field, as in Ref. [3], we first consider a radially polarized TEM₁₀ mode with the transverse field:

$$\mathbf{E}_\perp^L(\mathbf{r}, z, t) = E_0 e^{ik_L z - i\omega_L t} \frac{w_0}{w} \frac{\mathbf{r}}{w} \exp\left(-\frac{r^2}{w^2} - i\frac{k_L r^2}{2f} + 2i\psi\right), \quad (17)$$

where ω_L is the laser frequency, $k_L = \omega_L/c = 2\pi/\lambda_L$, the laser waist with a transverse size w_0 is assumed to be located at the screen, and hence

$$w^2 = w_0^2 \left(1 + \frac{z^2}{z_R^2}\right), \quad z_R = \frac{k_L w_0^2}{2},$$

$$f = z + \frac{z_R^2}{z}, \quad \psi = \arctan\left(\frac{z}{z_R}\right). \quad (18)$$

The choice of this higher-order mode is motivated in part by the fact that it matches the radial polarization of the diffraction radiation in Eq. (14) and is expected to produce better acceleration for the same laser energy.

Equation (10) (with the superscript “P” substituted for “L”) enables us to calculate the diffraction of the laser field through the round hole. First, we Fourier transform Eq. (17):

$$\begin{aligned} \mathcal{E}^L(\mathbf{r}, z, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{i\omega t} E_r^L(r, z, t) \\ &= E_0 \delta(\omega - \omega_L) e^{ik_L z} \frac{w_0}{w} \frac{\mathbf{r}}{w} \exp\left(-\frac{r^2}{w^2} - i\frac{kr^2}{2f} + 2i\psi\right). \end{aligned} \quad (19)$$

Putting this expression into Eq. (10) yields

$$\mathcal{E}_r^{LS}(z > 0) = E_0 \delta(\omega - \omega_L) \frac{e^{ik_R} (-k)}{R w_0} \int_0^a dr r^2 J_1(kr\theta) e^{-r^2/w_0^2}, \quad (20)$$

where, as was defined in Sec. II, the superscript “LS” stands for the diffracted laser field. Note that for $a \rightarrow \infty$, Eq. (20) can be integrated to yield

$$\begin{aligned} \mathcal{E}_r^{LS}(z > 0, a \rightarrow \infty) &= -E_0 \delta(\omega - \omega_L) \frac{e^{ikz} k^2 w_0^3 \theta}{z} \\ &\times \exp\left(-\frac{k^2 w_0^2 \theta^2}{4}\right), \end{aligned} \quad (21)$$

which is consistent with Eq. (19) in the limit $z \rightarrow \infty$.

As pointed out in Sec. II, it is convenient to represent the diffracted laser field as a sum of the original laser field (when the screen is absent) and the field \mathcal{E}^{LR} due to the radiation of the currents in the screen, $\mathcal{E}^{LS} = \mathcal{E}^L + \mathcal{E}^{LR}$. For the radiation field, we have

$$\mathcal{E}_r^{LR} = E_0 \delta(\omega - \omega_L) \frac{e^{ik_R} k}{R w_0} \int_a^\infty dr r^2 e^{-r^2/w_0^2} J_1(kr\theta). \quad (22)$$

This field was calculated in the region $z > 0$. However, due to the symmetry of the screen, it is symmetric about the point $z = 0$. In the region $z < 0$, θ is then taken to be the angle relative to the $(-z)$ axis; here \mathcal{E}_r^{LR} represents the reflected waves propagating in the direction opposite to the incident laser beam.

We now calculate the acceleration of this laser mode using the energy balance Eq. (8). Ignoring a pure phase factor, and noting that both fields \mathcal{E}^{LS} and \mathcal{E}^{PR} have radial polarization, the energy gain of the particle is

$$\Delta U_{\text{acc}} = c \left| \int_{-\infty}^{\infty} d\omega \int_S dS \mathcal{E}_r^{LS} (\mathcal{E}_r^{\text{PR}})^* \right|. \quad (23)$$

In the region $z < 0$, only the reflected field \mathcal{E}^{LR} can interfere with the radiation field since they propagate in the same direction (to the left of the screen in Fig. 1). Writing $\int_S dS = R^2 \int d\Omega = 2\pi R^2 \int_0^\infty \theta d\theta$ and inserting Eqs. (13), (20), and (22), we have

$$\begin{aligned} \Delta U_{\text{acc}} &= \frac{2\pi c E_0 k_L}{w_0} \left| \int_0^\infty \theta d\theta \frac{ek_L^2}{\pi \gamma c} \int_a^\infty r dr K_1\left(\frac{k_L r}{\gamma}\right) J_1(k_L r \theta) \int_0^a dr' (r')^2 e^{-(r')^2/w_0^2} J_1(k_L r' \theta) \right. \\ &\quad \left. - \int_0^\infty \theta d\theta \frac{ek_L^2}{\pi \gamma c} \int_a^\infty r dr K_1\left(\frac{k_L r}{\gamma}\right) J_1(k_L r \theta) \int_a^\infty dr' (r')^2 e^{-(r')^2/w_0^2} J_1(k_L r' \theta) \right|. \end{aligned} \quad (24)$$

Using the orthogonality of Bessel functions,

$$\int_0^\infty \theta d\theta J_\nu(kr\theta)J_\nu(kr'\theta) = \frac{\delta(kr - kr')}{kr}, \quad (25)$$

we obtain

$$\Delta U_{\text{acc}} = \frac{2eE_0k_L}{\gamma w_0} \int_a^\infty r^2 dr e^{-r^2/w_0^2} K_1\left(\frac{k_L r}{\gamma}\right). \quad (26)$$

For an ultrarelativistic particle we have $k_L \ll \gamma/w_0$ and we may use the approximation $K_1(x) \approx 1/x$. Introducing the laser focusing angle $\alpha_f = 2/(k_L w_0)$, the condition for the approximation can also be written as $\alpha_f \gg \gamma^{-1}$. Equation (26) then yields

$$\Delta U_{\text{acc}} \approx eE_0 w_0 e^{-a^2/w_0^2} = 4e\sqrt{\frac{2P_L}{c}} \exp\left(-\frac{a^2}{w_0^2}\right), \quad (27)$$

where the average power carried by this mode is

$$P_L = \frac{c}{8\pi} \int_{z=0 \text{ plane}} dS E_r^L H_r^L \\ = \frac{c}{8\pi} E_0^2 2\pi \int_0^\infty r dr \frac{r^2}{w_0^2} \exp\left(-\frac{r^2}{w_0^2}\right) = \frac{c}{32} E_0^2 w_0^2. \quad (28)$$

Equation (27) shows an important result: in order to accelerate a particle the laser beam should also irradiate the material wall of the screen. If the focal size of the laser light is so small that it does not touch the metal, $w_0 \ll a$, the acceleration diminishes exponentially. For optimal acceleration, we should have $a < w_0$ with the maximum energy gain in units of mc^2 :

$$\Delta \gamma_{\text{max}} = 4\sqrt{2} \sqrt{\frac{P_L}{P_0}}, \quad (29)$$

where $P_0 = m^2 c^5 / e^2 \approx 8.7$ GW. For a 1 TW laser, we find $\Delta \gamma_{\text{max}} \approx 60$.

In a general case of arbitrary relation between k_L and γ/w_0 , Eq. (26) can be rewritten as

$$\Delta \gamma_{\text{acc}} = 4\sqrt{2} \sqrt{\frac{P_L}{P_0}} G(A, B), \quad (30)$$

where

$$G(A, B) = 2B \int_A^\infty dx x^2 e^{-x^2} K_1(Bx), \quad A = \frac{a}{w_0}, \\ B = \frac{k_L w_0}{\gamma} = \frac{2}{\gamma \alpha_f}. \quad (31)$$

The maximum of G is 1 when $A = B = 0$. When $B = 0$, $G(A, 0) = e^{-A^2}$, which is the approximation used in Eq. (27). When $a = A = 0$,

$$G(0, B) = \left[1 - \frac{B^2}{4} \exp\left(\frac{B^2}{4}\right) \Gamma\left(0, \frac{B^2}{4}\right)\right], \quad (32)$$

where $\Gamma(0, Z) = \int_Z^\infty dt e^{-t}/t$ is the incomplete Gamma function. As shown in Appendix A, Eq. (32) agrees

with the direct integration of Eq. (1) in the absence of a hole, confirming the validity of this approach.

V. ACCELERATION BY TWO CROSSED LASER BEAMS

Another laser acceleration scheme employs a pair of linearly polarized laser beams with the Gaussian fundamental mode focused to the screen and crossed at a small angle to the z axis. If the two identical lasers are out of phase by π , the transverse components cancel while the longitudinal components add. In the absence of a beam-passage aperture, the acceleration has been directly calculated by integrating the longitudinal field along the beam trajectory [3]. Here we calculate the energy gain from the energy balance Eq. (8). It is sufficient to consider one tilted laser since the total energy gain of two crossed laser beams at a proper relative phase is twice as large.

First we calculate the laser field in the presence of the screen, following closely the derivation of Sec. IV. The Gaussian fundamental mode for a small tilt angle $\alpha \ll 1$ at the screen location, $z = 0$, is

$$\mathcal{E}_x^L(\mathbf{r}, z=0, \omega) \approx E_0 \delta(\omega - \omega_L) e^{ik_x \sin \alpha} \exp\left(-\frac{r^2}{w_0^2}\right). \quad (33)$$

The diffraction integral can be evaluated as [7]

$$\mathcal{E}_x^{LS}(z > 0) = E_0 \delta(\omega - \omega_L) \frac{-ike^{ikR}}{R} \int_0^a r dr \\ \times \exp\left(-\frac{r^2}{w_0^2}\right) J_0(kr\xi), \quad (34)$$

where $\xi = (\theta^2 + \alpha^2 - 2\theta\alpha \cos\phi)^{1/2}$, and ϕ is the azimuthal angle of the wave vector \mathbf{k} with respect to the z axis. In the region $z < 0$, the total laser field is the incident field and the reflected field given by

$$\mathcal{E}_x^{LR} = E_0 \delta(\omega - \omega_L) \frac{ike^{ikR}}{R} \int_a^\infty r dr \exp\left(-\frac{r^2}{w_0^2}\right) J_0(kr\xi). \quad (35)$$

To compute Eq. (8), we note that $|\mathcal{E}^{LS} \cdot \mathcal{E}^{BS*}| = |\mathcal{E}_x^{LS} \mathcal{E}_r^{BR} \cos\phi|$ and make use of the Bessel function expansion [12]

$$J_0(kr\xi) = \sum_{m=-\infty}^{\infty} J_m(kr\theta) J_m(kr\alpha) e^{im\phi}. \quad (36)$$

Integration over ϕ picks up only $m = \pm 1$ terms. Then following the integration steps of Sec. IV, we find

$$\Delta U_{\text{acc}} = 2 \frac{2eE_0k_L}{\gamma} \int_a^\infty r dr K_1\left(\frac{k_L r}{\gamma}\right) \exp\left(-\frac{r^2}{w_0^2}\right) J_1(k_L r \alpha) \\ \approx 4eE_0 w_0 \int_{a/w_0}^\infty dx e^{-x^2} J_1\left(\frac{2\alpha}{\alpha_f} x\right), \quad (37)$$

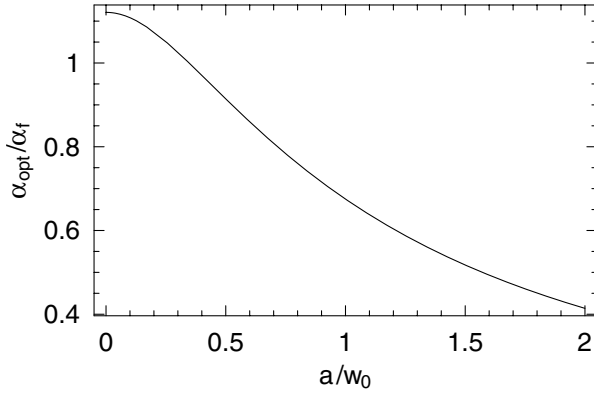


FIG. 2. The optimal tilt angle as a function of the hole radius for the tilted Gaussian laser beam.

where the extra factor of 2 on the right-hand side takes into account two crossed laser beams, and we have assumed that $k_L \ll \gamma/w_0$ to use $K_1(x) \approx 1/x$ for the approximate expression. For a vanishing hole as $a \rightarrow 0$, we have

$$\Delta U_{\text{acc}} = 2eE_0w_0 \frac{\alpha_f}{\alpha} \left[1 - \exp\left(-\frac{\alpha^2}{\alpha_f^2}\right) \right], \quad (38)$$

in agreement with Ref. [3] when the injection point is at $z_I = -\infty$ and the extraction point is at $z_F = 0$. At the optimal tilt angle $\alpha_{\text{opt}} \approx 1.1\alpha_f$, the maximum energy gain is $1.3eE_0w_0$. For an arbitrary a , Eq. (37) can be used to obtain the optimal tilt angle and the maximum energy gain (see Figs. 2 and 3). As shown in Fig. 3, the maximum energy gain in units of mc^2 can be approximated by

$$\Delta \gamma_{\text{max}} \approx 3.6 \sqrt{\frac{P_L}{P_0}} \exp\left(-\frac{a^2}{w_0^2}\right). \quad (39)$$

Here $P_L = cE_0^2w_0^2/8$ is the total laser power for the two

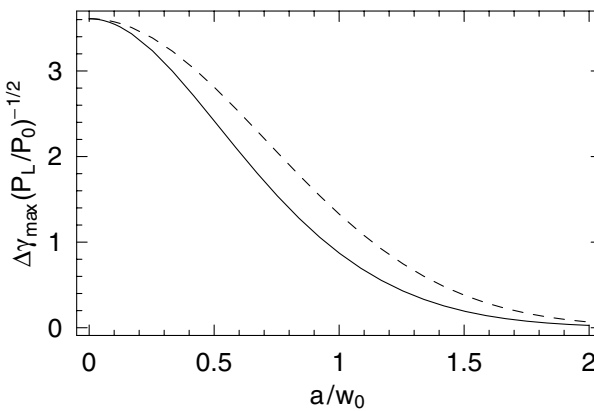


FIG. 3. The maximum energy gain of two crossed laser beams evaluated at the optimal tilt angle from Eq. (37) (solid line), and compared with the approximate Eq. (39) (dashed line).

Gaussian beams. Comparing with Eq. (27), the energy gain of the crossed lasers has essentially the same exponential dependence on the radius of the hole. For the same laser power, the radially polarized TEM_{10} mode is more effective for acceleration (by about a factor of 1.6) because it matches the polarization of the diffraction radiation in this accelerating structure (see Sec. VI B for more discussions).

VI. DISCUSSION

A. Limitations due to material damage

Results of previous sections suggest that the laser should irradiate the accelerator structure, which is subject to material damage at a certain threshold laser fluence. Considering the case of the higher-order laser mode in Sec. IV, we rewrite Eq. (30) as

$$\begin{aligned} \Delta \gamma_{\text{acc}} &= 4\sqrt{2} \left(\frac{U_L}{w_0^2 P_0 t_L} \right)^{1/2} w_0 G(A, B) \\ &\approx 2 \left(\frac{F_L}{P_0 t_L} \right)^{1/2} \gamma \lambda_L B G(A, B), \end{aligned} \quad (40)$$

where U_L is the laser flash energy, t_L is the laser pulse duration, and $F_L \approx 0.24U_L/w_0^2$ is the maximum laser fluence at $r \approx 0.7w_0$ for this higher-order mode laser. We have also assumed the hole radius $a < 0.7w_0$ for effective acceleration and used the previous notations $A = a/w_0$ and $B = k_L w_0/\gamma = 2/(\gamma \alpha_f)$ from Eq. (31). Since the laser fluence at the material damage threshold is known to be $F_{\text{th}} \approx 2 \text{ J/cm}^2$ for sub-ps laser pulses [14], we assume that the laser operates at the damage threshold [i.e., by taking $F_L = F_{\text{th}}$ in Eq. (40)] and optimize the laser spot size or the focusing angle in order to obtain the maximum energy gain. Figure 4 shows the optimal focusing angle that maximizes $BG(A, B)$ in Eq. (40). For $a \ll w_0$, the optimal laser focusing angle $(\alpha_f)_{\text{opt}} \approx \gamma^{-1}$ and the optimal spot size $(w_0)_{\text{opt}} \approx \gamma \lambda_L/\pi$.

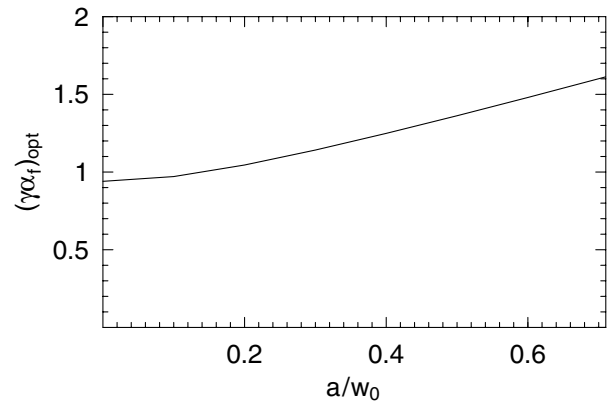


FIG. 4. The optimal laser focusing angle as a function of the hole radius at the material damage threshold.

The scaling $(\alpha_f)_{\text{opt}} \approx \gamma^{-1}$ has a simple physical explanation. As it follows from Eq. (27), in the limit of large angles $\alpha_f \gg \gamma^{-1}$, the energy gain does not depend on the spot size w_0 and scales with the laser power as $P_L^{1/2}$. This happens because a particle interacts with the laser on the Rayleigh length, and although, for a given P_L , increasing w_0 makes the amplitude of the laser field smaller, a corresponding increase in the Rayleigh length compensates for the smaller field and makes the energy gain independent on w_0 . Further increasing w_0 , however, makes the angle α_f smaller than γ^{-1} . In this regime, the interaction length becomes shorter than the Rayleigh length—it is determined by the phase slippage due to the difference between the particle's velocity and the phase velocity of the laser light. As it follows from Eq. (30), the energy gain in this regime (for a given P_L) decreases with w_0 . For a given fluence (laser power per unit area) the optimal value of α_f turns out to be at the boundary between those two regimes.

At the optimal focusing angle $(\alpha_f)_{\text{opt}} \approx \gamma^{-1}$, we have $B_0 = 2/[\gamma(\alpha_f)_{\text{opt}}] \approx 2$ in Eq. (40) and $B_0 G(A, B_0) \approx 0.8$ for $a \ll w_0$ from Fig. 5. If we take a typical short-pulse laser with $t_L \approx 100$ fs and $\lambda_L \approx 1 \mu\text{m}$, the maximum fractional energy gain limited by the fluence damage threshold is approximately

$$\frac{\Delta\gamma_{\text{max}}}{\gamma} \approx 1.6\lambda_L \sqrt{\frac{F_{\text{th}}}{P_0 t_L}} \approx 7.5 \times 10^{-3}. \quad (41)$$

Since the interaction length is about equal to the Rayleigh length $z_R = \pi(w_0)_{\text{opt}}^2/\lambda_L \approx \gamma^2 \lambda_L/\pi$ at the optimal spot size, the effective acceleration gradient is

$$\frac{\Delta U_{\text{acc}}}{z_R} \approx \frac{7.5 \times 10^{-3} \gamma m c^2}{\gamma^2 \lambda_L/\pi} \approx \frac{12}{\gamma} \text{ GeV/m}. \quad (42)$$

For a 50 MeV electron (i.e., $\gamma \approx 100$), the energy gain is about 375 keV from Eq. (41), and the acceleration gradient

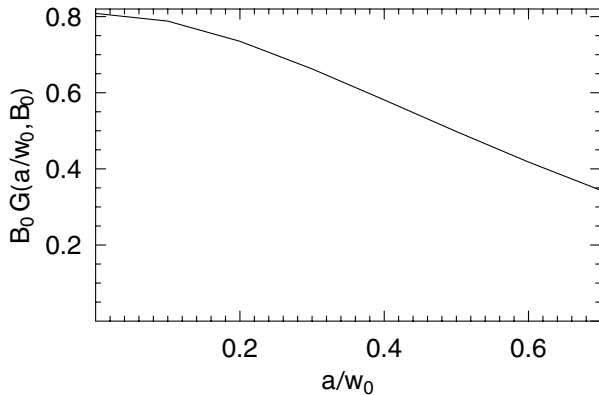


FIG. 5. The function $B_0 G(A, B_0)$ in Eq. (40) evaluated at the optimal spot size or focusing angle.

is about 120 MeV/m according to Eq. (42), in agreement with the expected performance of the E-163 proposal [5].

Finally, suppose that the laser is operated at the damage threshold fluence, the optimal laser energy at the optimal spot size for maximum energy gain is

$$U_L \approx \frac{F_{\text{th}}(w_0)_{\text{opt}}^2}{0.24} \approx 8.4\gamma^2 \text{ nJ}. \quad (43)$$

For a 100 fs laser pulse interacting with a 50 MeV electron beam, the optimal laser power is $P_L \approx 0.84$ GW. A larger laser power at the given pulse duration requires a larger laser spot to avoid the material damage and hence a smaller longitudinal field for acceleration. Since the interaction length is still $\sim \gamma^2 \lambda_L/\pi$ limited by the phase slippage of the particle in the laser field, the total energy gain is actually smaller if the laser power is larger than this optimal value.

B. Optimal laser profile

As pointed out in Sec. V, the radially polarized TEM_{10} mode is more effective for laser acceleration than the tilted Gaussian fundamental mode because it matches better with the diffraction radiation pattern. For optimal acceleration, one might consider shaping the laser transverse profile in such a way that achieves maximum acceleration for a given laser power. It is easy to see from calculations in Sec. IV that for the optimum acceleration the angular distribution of the reflected laser light \mathcal{E}^{LR} must match exactly the angular distribution of the particle's radiation. In the case $a \ll w_0$, this means [see Eq. (16)]

$$\mathcal{E}_r^{\text{LR}} = E_0 w_0 \delta(\omega - \omega_L) \frac{e^{ikz}}{z} \frac{\theta}{\theta^2 + \gamma^{-2}}. \quad (44)$$

The corresponding laser power is $P_L \approx (\ln \gamma) c E_0^2 w_0^2/4$, for $\gamma \gg 1$. Integrating the energy balance Eq. (8) then yields

$$\Delta\gamma_{\text{max}} = (2 \ln \gamma) e E_0 w_0 = 4\sqrt{\ln \gamma} \sqrt{\frac{P_L}{P_0}}. \quad (45)$$

We see that the optimal laser profile (with the angular distribution of the transition radiation) only improves the maximum energy gain by a small factor $\sim \sqrt{\ln \gamma}$ even for an ultrarelativistic particle.

VII. CONCLUSION

In summary, linear acceleration by a laser field in vacuum is possible only if a particle radiates in passing the accelerating structure. In this paper, we express the energy gain by the particle as an interference integral of

the laser field and the radiation field in the far zone and hence avoid calculation of any near field that accelerates the particle. We apply this new method to study laser acceleration in a simple accelerating structure (a conducting screen with a beam-passing hole) and to optimize gain for given laser parameters. We show that for optimal acceleration, the laser should irradiate on the accelerating structure (i.e., the dimension of the hole should be less than the laser spot size), and the laser focusing angle (as well as the crossing angle in the case of the two crossed laser beams) should be comparable to the radiation opening angle γ^{-1} . Limited by the damage threshold fluence, the maximum energy gain in this accelerating structure is proportional to the electron energy, but the acceleration gradient scales as γ^{-1} .

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APPENDIX: COMPARISON WITH DIRECT CALCULATION OF ACCELERATION

In the case with no hole, $a = 0$, when the screen stops the laser beam from propagating to the region $z > 0$ the energy gain in a laser field can be calculated directly. Particles are accelerated by the laser beam in the region $z < 0$ and stop interacting with the laser beam after passing through the screen. (In the absence of the screen, the laser would decelerate electrons in the region $z > 0$ so that the net energy gain is zero.)

For the laser mode given by Eq. (17), the longitudinal electric field can be found from the Maxwell equation $\nabla \cdot \mathbf{E}^L = 0$ and is approximated by

$$\begin{aligned} E_z^L &\approx \frac{i}{k_L} \nabla_{\perp} \cdot \mathbf{E}_{\perp}^L = \frac{i}{k_L} \frac{1}{r} \frac{\partial}{\partial r} (r E_r^L) \\ &= E_0 e^{ik_L z - i\omega_L t} \frac{w_0}{w} \left[\frac{i}{k_L} \frac{2}{w} \left(1 - \frac{r^2}{w^2} \right) + \frac{r^2}{wf} \right] \\ &\quad \times \exp \left(-\frac{r^2}{w^2} - i \frac{k_L r^2}{2f} + 2i\phi \right). \end{aligned} \quad (\text{A1})$$

Consider a relativistic particle moving in the z direction along the axis of the system, $\mathbf{r} = 0$ and $z = vt$. The energy gain can be obtained by integrating the longitudinal laser field along the particle's trajectory from $z = -\infty$ to $z = 0$ (the location of the screen):

$$\begin{aligned} \Delta U_{\text{acc}} &= \int_{-\infty}^0 dz e E_z^L(\mathbf{r} = 0, t = z/v) \\ &= \frac{2eE_0}{k_L w_0} \int_{-\infty}^0 dz \frac{i}{(1 - iz/Z_R)^2} \exp \left[ik_L z \left(1 - \frac{c}{v} \right) \right] \\ &= eE_0 w_0 \left[1 - \frac{B^2}{4} \exp \left(\frac{B^2}{4} \right) \Gamma \left(0, \frac{B^2}{4} \right) \right], \end{aligned} \quad (\text{A2})$$

where $B = k_L w_0 / (\gamma)$, and the square bracket term describes the gain reduction due to relative slippage of the particle in the laser field. This expression is identical to Eq. (32) derived using the energy balance Eq. (8). An approximate expression of Eq. (A2) is given in Ref. [3].

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- [1] R. B. Palmer, in *Advanced Accelerator Concepts*, edited by P. Schoessow, AIP Conf. Proc. No. 335 (AIP, New York, 1995), pp. 90–100.
 - [2] F. Caspers and E. Jensen, CERN Technical Report No. CERN/PS 89-69, 1989.
 - [3] E. Esarey, P. Sprangle, and J. Krall, Phys. Rev. E **52**, 5443 (1995).
 - [4] R. Byer, T. Plettner, C. Barnes, E. Colby, B. Cowan, R. Siemann, and J. Spencer, in *Proceedings of the 2001 Particle Accelerator Conference* (IEEE, Piscataway, NJ, 2001).
 - [5] C. Barnes, E. Colby, B. Cowan, R. Noble, D. Palmer, R. Siemann, J. Spencer, and D. Walz, SLAC Technical Report No. E-163, 2001.
 - [6] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Course of Theoretical Physics Vol. 2 (Pergamon, London, 1979), 4th ed. (translated from the Russian).
 - [7] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed.
 - [8] M. Zolotarev, S. Chattopadhyay, and K. McDonald, <http://viper.hep.princeton.edu/~mcdonald/accel/vacuum-accel.pdf>
 - [9] M. Xie, in *Proceedings of the 2003 Particle Accelerator Conference* (IEEE, Piscataway, NJ, 2003).
 - [10] M. Ter-Mikaelian, *High-Energy Electromagnetic Processes in Condensed Media* (Wiley-Interscience, London, 1972).
 - [11] B. M. Bolotovskii and E. A. Galst'yan, Phys. Usp. **43**, 755 (2000).
 - [12] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, New York, 2000), 6th ed.
 - [13] G. Dome, E. Gianfelice, L. Palumbo, V. G. Vaccaro, and L. Verolino, Nuovo Cimento Soc. Ital. Fis. **104A**, 1241 (1991).
 - [14] B. Stuart, M. Feit, A. Rubenchik, B. W. Shore, and M. Perry, Phys. Rev. Lett. **74**, 2248 (1995).